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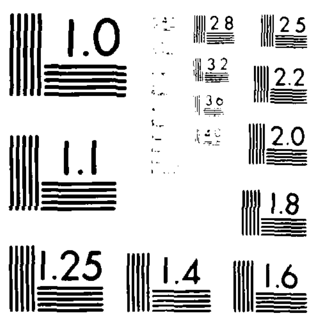

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QUASISTABLE PARAMETRIC OPTIMIZATION WITHOUT COMPACT LEVEL SETS

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ABSTRACT

The well-known perturbational duality theory for convex optimization is refined to handle directly, in locally convex Hausdorff spaces, problems involving noncoercive convex functionals together with unbounded densely defined linear operators or, more generally, convex processes. The theory presented includes conjugacy, recession, and ε-subdifferential formulas for the two fundamental pairs of dual operations and also includes systematic treatment of ε-solutions.

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Key Words: Convex optimization, noncoercive functionals, unbounded operators, convex processes, ε-solutions, weak local compactness, recession theory, dual operations, ε-subdifferentials, saddlepoint problems, perturbational duality, abstract Hamiltonian system, abstract Euler-Lagrange system

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SIGNIFICANCE AND EXPLANATION

Dual variational principles, which arise classically in physics and mechanics and typically involve partial differential operators, usually correspond to dual pairs of constrained convex optimization problems in various infinite-dimensional function space settings. Constrained optimization denotes the search for an optimum, say a minimum, of some given criterion functional over a set of candidate solutions which satisfy given side conditions, or constraints. Convex problems form the simplest class of nonlinear problems. Dual pairs of convex optimization problems are nonlinear generalizations of the familiar, widely useful dual pairs of linear programming problems. Dual pairs of constrained convex optimization problems arise also in fields other than physics and mechanics, including mathematical economics, operations research, management science, many subfields of engineering, and mathematics itself.

The mathematical model for constrained convex optimization problems known as perturbational duality theory, developed some 10 to 15 years ago, provides a nearly complete mathematical treatment of those basic situations which involve "nice" (i.e. coercive) convex functionals and "nice" (i.e. bounded) linear operators. Much less developed is the treatment for problems involving noncoercive functionals and unbounded operators only densely defined. Even less understood are problems involving more complicated operators called convex processes, which in general are multivalued and not even densely defined, and which are important in problems from, e.g., mathematical economics.

This paper presents a mathematical theory, valid in very general spaces, for dual pairs of constrained convex optimization problems involving noncoercive functionals and unbounded linear operators or, more generally, convex processes. It constitutes a fundamental refinement of the well-known perturbational duality model cited above. Systematic treatment of approximate optimal solutions is also provided, rendering the model more amenable to further, numerical analysis.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

QUASISTABLE PARAMETRIC OPTIMIZATION WITHOUT COMPACT LEVEL SETS

L. McLinden

It is well known that a lower semicontinuous (l.s.c.) function $\Phi : X \to (-\infty,\infty]$, $\Phi \not\equiv \infty$, achieves a finite infimum if the sets $\{x|\Phi(x) \le \alpha\}$, $\alpha \in \mathbf{R}$, are compact. Convex conjugate duality establishes that, if Φ is convex, a dual sufficient condition for such compactness is that the conjugate function Φ^* be continuous at 0. Much work in constrained optimization has dealt with elaborating and implementing this result (see References). The present work is rooted in the following generalization: a direct sufficient condition for such a convex Φ to achieve a finite infimum is that (i) $\{x|\Phi(x) \le \alpha\}$, $\alpha \in \mathbf{R}$, are locally compact and (ii) Φ is constant along the whole line generated by any direction in which Φ does not eventually increase. Here we present theorems embodying this result in strengthened, augmented form tailored to basic structure occurring widely in constrained optimization. They go well beyond the closest counterparts now available (including, notably, [5], [10], [9], [1, Ch. 14]).

A word on notation. All spaces are locally convex Hausdorff topological vector spaces over $\mathbf{R}$; X, V are paired in duality, as are U, Y. For a multifunction T from Z into W (i.e. $T : Z \to 2^W$), $G(T) := \{(z,w)|w \in T(z)\}$, $D(T) := \{z|T(z) \neq \emptyset\}$, $R(T) := \{w|\exists z, w \in T(z)\}$. For T and any $\Phi : Z \to [-\infty,\infty]$, define $T\Phi$ on W by $(T\Phi)(w) := \inf\{\Phi(z)|z \in T^{-1}(w)\}$, with $\inf \emptyset := \infty$. For Φ l.s.c. proper convex, $\Phi 0^+(z) := \sup\{\lambda^{-1}(\Phi(\bar{z} + \lambda z) - \Phi(\bar{z}))|0 < \lambda < \infty\}$ for any fixed $\bar{z} \in \operatorname{dom} \Phi := \{z|\Phi(z) < \infty\}$, and if Z is paired in duality with W, $\Phi^*(w) := \sup\{\langle z,w\rangle - \Phi(z)|z \in Z\}$ and $G(\partial_\varepsilon \Phi) := \{(z,w)|\Phi^*(w) \le \varepsilon + \langle z,w\rangle - \Phi(z)\}$ for each fixed $\varepsilon \in [0,\infty)$. We abbreviate "weakly locally compact" by w.l.c.

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THEOREM I. Let h be l.s.c. proper convex on X with $k := h^*$. Let A be a densely defined linear operator from X into U with closed graph and adjoint B. Define kB on Y by $(kB)(y) := k(By)$ if $y \in D(B)$ and $:= \infty$ if $y \notin D(B)$. Assume (i) $D(A) \cap \operatorname{dom} h$ is nonempty and w.l.c. and (ii) $x \in A^{-1}(0)$, $h0^+(x) \leq 0 \implies h0^+(-x) \leq 0$. Then (1) $(kB)^*(u) = (Ah)(u) > -\infty$ and (2) $(Ah)0^+(u) = (A(h0^+))(u) > -\infty$, with both infima on the right attained whenever not ∞ vacuously. For any $\varepsilon \in [0,\infty)$, $y \in \partial_\varepsilon(Ah)(u) \iff u \in \partial_\varepsilon(kB)(y) \iff y \in D(B)$, $u \in A\partial_\varepsilon k(By)$.

Notice (1) $\implies \emptyset \neq R(B) \cap \operatorname{dom} k$. Further, by (1), (2) is equivalent to:

$$(2') \qquad \sup_{y \in B^{-1}\operatorname{dom} k} \langle u,y\rangle = \inf_{x \in A^{-1}(u)} \sup_{v \in \operatorname{dom} k} \langle x,v\rangle > -\infty ,$$

with the infimum attained whenever $u \in R(A)$. If (ii) holds in the form "$x \in A^{-1}(0)$, $h0^+(x) \leq 0 \implies x = 0$," the infimum problems on the right of (1), (2), (2') have weakly compact ε-approximate solution sets for all $\varepsilon \in [0,\infty)$. "Quasistable" in the title denotes the fact that, whenever $(Ah)(u) < \infty$ in (1), one can prove the u.s.c. proper concave $q(v) := \inf\{h(x) - \langle x,v\rangle \mid x \in A^{-1}(u)\}$ on V is Mackey quasicontinuous [5] and that the "inf" defining q is attained for all v (e.g., $v = 0$) in the Mackey relative interior of $\{v \mid q(v) > -\infty\}$. Counterparts of these remarks apply below.

THEOREM II. For $j = 1,\ldots,m$ let f_j be l.s.c. proper convex on X with $g_j := f_j^*$. Assume (i) $\operatorname{dom} f_2,\ldots,\operatorname{dom} f_m$ are w.l.c. and (ii) $x_1 + \ldots + x_m = 0$, $f_1 0^+(x_1) + \ldots + f_m 0^+(x_m) \leq 0 \implies f_1 0^+(-x_1) + \ldots + f_m 0^+(-x_m) \leq 0$. Then (1) $(g_1 + \ldots + g_m)^*(x) = (f_1 \,\square \ldots \square\, f_m)(x) > -\infty$, and (2) $(f_1 \,\square \ldots \square\, f_m)0^+(x) = (f_1 0^+ \,\square \ldots \square\, f_m 0^+)(x) > -\infty$, with both infima on the right attained. [$\square$ denotes inf-convolution.] For any $\varepsilon \in [0,\infty)$, $v \in \partial_\varepsilon(f_1 \,\square \ldots \square\, f_m)(x) \iff x \in \partial_\varepsilon(g_1 + \ldots + g_m)(v) \iff x \in \cup\{\partial_{\varepsilon_1} g_1(v) + \ldots + \partial_{\varepsilon_m} g_m(v) \mid \varepsilon_j \geq 0, \varepsilon_1 + \ldots + \varepsilon_m = \varepsilon\}$.

THEOREM III. Let h be l.s.c. proper convex on X with $k := h^*$. Let A be a closed convex process [8, §39] from X into U (i.e. $G(A)$ is a nonempty closed convex cone) with adjoint A^* from Y into V given by $v \in A^*(y) \iff \langle u,y\rangle \le \langle x,v\rangle$ for all $u \in A(x)$. Assume (i) $\emptyset \neq D(A) \cap \operatorname{dom} h$ and $\operatorname{dom} h$ is w.l.c. and (ii) $x \in A^{-1}(0)$, $h0^+(x) \le 0 \Longrightarrow h0^+(-x) \le 0$. Then, for $B := A^{*-1}$, (1) $(Bk)^*(u) = (Ah)(u) > -\infty$ and (2) $(Ah)0^+(u) = (A(h0^+))(u) > -\infty$, with both infima on the right attained whenever not ∞ vacuously. Further, $y \in \partial_o(Ah)(u) \iff$ $u \in \partial_o(Bk)(y) \iff [\exists x \in A^{-1}(u)\ \exists v \in B^{-1}(y)$ such that $\langle u,y\rangle \ge \langle x,v\rangle$, $v \in \partial_o h(x)]$. For any $\varepsilon,\delta \in [0,\infty)$ and $(v,y) \in G(B)$ with $k(v) \le \delta + (Bk)(y) < \infty$, $y \in \partial_\varepsilon(Ah)(u) \iff u \in \partial_\varepsilon(Bk)(y) \Longrightarrow [u \in R(A)$ and $\exists x \in A^{-1}(u)\ \exists \alpha \ge 0\ \exists \beta \ge 0$ such that $\alpha + \beta = \varepsilon + \delta$, $\langle u,y\rangle \ge \langle x,v\rangle - \beta$, $x \in \partial_\alpha k(v)] \Longrightarrow u \in \partial_{\varepsilon+\delta}(Bk)(y) \iff$ $y \in \partial_{\varepsilon+\delta}(Ah)(u)$.

THEOREM IV. Let F be l.s.c. proper convex on $X \times U$. Let A be a linear transformation from X into U with either A continuous or A densely defined with closed graph. In the latter case, assume $[F(\cdot,u) \not\equiv \infty \Longrightarrow F(\cdot,u)$ somewhere continuous], or dually, $[F^*(v,\cdot) \not\equiv \infty \Longrightarrow F^*(v,\cdot)$ somewhere continuous]. Assume (i) $\{x \in D(A) \mid \exists u,\ F(x,u) < \infty\}$ is nonempty and w.l.c. and (ii) $x \in D(A)$, $F0^+(x,Ax) \le 0 \Longrightarrow F0^+(-x,-Ax) \le 0$. Then

$$-\infty < \min_{x \in D(A)} F(x,u + Ax) = \sup_{y \in D(A^*)} \{-F^*(A^*y,-y) - \langle u,y\rangle\} =: p(u),$$

$$-\infty < \min_{x \in D(A)} \sup_{(y,v) \in \Delta} \langle (x,u),(v,-y)\rangle = \sup_{y \in \Delta_o} \langle u,-y\rangle = p0^+(u)\ ,$$

where $\Delta := \{(y,v) \mid y \in D(A^*),\ F^*(A^*y,-y) < \infty\}$, $\Delta_o := \{y \mid (y,0) \in \Delta\}$. For any $\varepsilon \in [0,\infty)$, $-y \in \partial_\varepsilon p(u) \iff y \in \operatorname{Arg}\ \varepsilon\text{-}\max\{-F^*(A^*\hat{y},-\hat{y}) - \langle u,\hat{y}\rangle \mid \hat{y} \in D(A^*)\} \iff$ $[y \in D(A^*)$ and $\exists x \in D(A)$ such that $(A^*y,-y) \in \partial_\varepsilon F(x,u + Ax)]$, and any such x belongs to $\operatorname{Arg}\ \varepsilon\text{-}\min\{F(\hat{x},u + A\hat{x}) \mid \hat{x} \in D(A)\}$. If also (iii) Δ_o is w.l.c. then, for each u satisfying $[y \in D(A^*),\ F^*0^+(A^*y,-y) + \langle u,y\rangle \le 0 \Longrightarrow F^*0^+(-A^*y,y) - \langle u,y\rangle \le 0]$, one has $p(u) \in \mathbb{R}$ and the "sup" defining $p(u)$ is actually "max."

THEOREM V. Let H be closed proper convex-concave [9] on $X \times Y$, put $F(x,u) := \sup\{H(x,y) - \langle u,y\rangle \mid y \in Y\}$, and assume this F together with A satisfy the hypotheses of Theorem IV. Then

$$-\infty < \min_{x\in D(A)} \sup_{y}\{H(x,y) - \langle Ax,y\rangle - \langle u,y\rangle\}$$

$$= \sup_{y\in D(A^*)} \inf_{x}\{H(x,y) - \langle x,A^*y\rangle - \langle u,y\rangle\} =: p(u) ,$$

$$-\infty < \min_{x\in D(A)} \sup_{(y,v)\in\Delta} \langle (x,u),(v,-y)\rangle = \sup_{y\in\Delta_o} \langle u,-y\rangle = p0^+(u) ,$$

where $\Delta := \{(y,v)|y \in D(A^*),\ G(y,v + A^*y) > -\infty\}$, $\Delta_o := \{y|(y,0) \in \Delta\}$ for $G(y,v)$ $:= \inf\{H(x,y) - \langle x,v\rangle | x \in X\}$. For any $\varepsilon \in [0,\infty)$, $-y \in \partial_\varepsilon p(u)$ <==>

$$y \in \operatorname{Arg}\ \varepsilon\text{-}\max_{\hat{y}\in D(A^*)} \inf_{\hat{x}}\{H(\hat{x},\hat{y}) - \langle \hat{x},A^*\hat{y}\rangle - \langle u,\hat{y}\rangle\}$$

$$\Longleftrightarrow \begin{cases} y \in D(A^*) \text{ and } \exists x \in D(A) \text{ such that} \\ (*) \quad \sup_{\hat{y}}\{H(x,\hat{y}) - \langle Ax,\hat{y}\rangle - \langle u,\hat{y}\rangle\} \le \varepsilon + \inf_{\hat{x}}\{H(\hat{x},y) - \langle \hat{x},A^*y\rangle - \langle u,y\rangle\} \end{cases}$$

<==> $y \in D(A^*)$ and $\exists x \in D(A)$ with (**) $(A^*y,-y) \in \partial_\varepsilon F(x,u + Ax)$.

If also (iii) Δ_o is w.l.c. then, for each u such that

$$\bigcap_{x\in C\cap D(A)} \{y|\bar{H}(x,\cdot)0^+(y) - \langle u + Ax,y\rangle \ge 0\} \text{ is a subspace,}$$

where $C := \{x|\bar{H}(x,\cdot)$ is somewhere finite$\}$ and $\bar{H}(x,\cdot)(y) := \operatorname{usc} H(x,\cdot)(y)$, one has $p(u) \in \mathbf{R}$ and the "sup" defining $p(u)$ is actually "max."

Relation (*) (resp. (**)) for $\varepsilon = 0$ can be regarded as the abstract Hamiltonian (resp. Euler-Lagrange) system for the setting of Theorems V, IV (cf. [1]).

Main outlines of proof. (I) follows from (IV) by replacing A by $-A$ and putting $F(x,u) := h(x) + \psi_{\{o\}}(u)$, where generally $\psi_S := 0$ on S and $:= \infty$ off S. (V) is deduced from (IV) plus saddle function theory. (IV) splits into two cases: that of A continuous is proved by refining for the map $(x,u) \to u$ the proof of (III) sketched below; that of A densely defined is deduced by applying the first to $F_1(x,u) := F(x,u + Ax) + \psi_{D(A)}(x)$. Here, one proves as preliminary facts formulas for $F_1 0^+$, F_1^*, $\partial_\varepsilon F_1$. (II) involves two steps: the case $m = 2$ is proved by refining for the map $(x_1,x_2) \to x_1 + x_2$ the proof of (III); then the continuous case of (IV) permits replacing f_2 by $f_2 \,\square \ldots \square\, f_m$.

Sketch for (III). Observe $(Bk)^* \leq Ah$ always. Let $(Bk)^*(\bar{u}) < \infty$. Define a l.s.c. proper convex F by $F(w,w_1,w_2) := g_1(w - w_1) + g_2(w - w_2) - \langle w,\bar{z}\rangle$, for $W = W_1 = W_2 = V \times Y$, $Z = Z_1 = Z_2 = X \times U$, $\bar{z} = (0,\bar{u})$, $g_1 = \psi_{G(B)}$, $g_2 = k + \psi_Y$. One shows $\alpha := \inf_w F(w,0,0)$ and $\beta := \inf_{z_1,z_2} F^*(0,z_1,z_2)$ satisfy $\alpha = -\beta$, with inf in β attained, if for $p(w_1,w_2) := \inf_w F(w,w_1,w_2)$ one has (a) $\alpha > -\infty$, (b) $(0,0)$ and $\operatorname{dom} p$ cannot be properly separated, and (c) $\exists$ l.s.c. proper convex $\rho \geq p$ such that $\{(z_1,z_2) \mid \rho^*(z_1,z_2) \leq \gamma\}$, $\gamma \in \mathbf{R}$, are w.l.c. Now $\alpha = -(Bk)^*(\bar{u})$ $\Rightarrow$ (a), and (ii) $\Leftrightarrow$ (b). For (c), take $\rho(w_1,w_2) := F(\bar{w} + w_1,w_1,w_2)$ for fixed $\bar{w} \in \operatorname{dom} g_1$. Then $\rho(w_1,w_2) = g_2(\bar{w} + w_1 - w_2) - \langle w_1,\bar{z}\rangle + c$, $c := g_1(\bar{w}) - \langle\bar{w},\bar{z}\rangle$, yields $\rho^*(z_1,z_2) = g_2^*(z) - \langle\bar{w},z\rangle - c$ if $(\bar{z},0) + (z_1,z_2) = (z,-z)$ for some $z \in Z$, and $\rho^* = \infty$ otherwise. Then (i) $\Rightarrow$ (c). Since $F^*(z,z_1,z_2) = h(-x_2)$ if $z = (x,u)$ and $z_i = (x_i,u_i)$ satisfy $z + z_1 + z_2 = \bar{z}$, $(x_1,-u_1) \in G(A)$, $u_2 = 0$, and $F^* = \infty$ otherwise, $\beta = (Ah)(\bar{u})$. Thus, (1) holds. The ∂_ε assertions are proved with the aid of (1). For (2), observe epi Ah lies between $(A \times I)\operatorname{epi} h$ and its closure, where I is the identity on $\mathbf{R}$; similarly for epi $A(h0^+)$. Using (i), (ii) and T, $G(T) := G(A \times I) \cap (\operatorname{epi} h \times U \times \mathbf{R})$, one proves $R(T) = (A \times I)\operatorname{epi} h$ closed. Then, using (i), (ii) and S, $G(S) := G(I \times A \times I) \cap (\operatorname{cl} H \times \mathbf{R} \times U \times \mathbf{R})$, where $H := \{\alpha(1,x,\xi) \mid \alpha > 0,\ (x,\xi) \in \operatorname{epi} h\}$, one proves $R(S) = \{(\sigma,u,\mu) \mid \sigma > 0,\ (u,\mu) \in (A \times I)(\sigma \operatorname{epi} h)\} \cup \{(0,u,\mu) \mid (u,\mu) \in (A \times I)0^+\operatorname{epi} h\}$ is closed. Since $J := \{\sigma(1,u,\mu) \mid \sigma > 0,\ (u,\mu) \in (A \times I)\operatorname{epi} h\}$ satisfies $J \subset R(S) \subset \operatorname{cl} J = J \cup \{(0,u,\mu) \mid (u,\mu) \in 0^+(A \times I)\operatorname{epi} h\}$, $R(S) = \operatorname{cl} J$ follows, whence

$(A \times I)0^+\text{epi } h = 0^+(A \times I)\text{epi } h$. The nontrivial half of this plus (ii) imply $(A \times I)\text{epi } h$ contains no "vertical" line, completing (2).

Details and related results will appear elsewhere.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The well-known perturbational duality theory for convex optimization is refined to handle directly, in locally convex Hausdorff spaces, problems involving noncoercive convex functionals together with unbounded densely defined linear operators or, more generally, convex processes. The theory presented includes conjugacy, recession, and ε-subdifferential formulas for the two fundamental pairs of dual operations and also includes systematic treatment of ε-solutions.

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